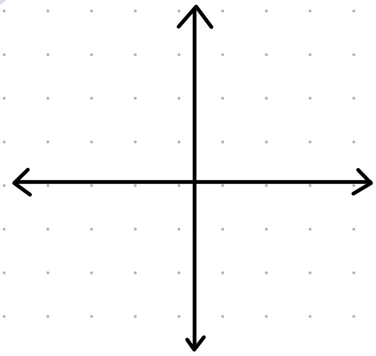
MATHS

maths



LINEAR REVIEW

① Calculate the distance between:

a) (0,3) and (6,9)

$d = \sqrt{(x_2 - x_1)^2 + (y_2 + y_1)^2}$

$d = \sqrt{(6-0)^2 + (9-3)^2}$

$d = 8.49$ units

b) (-4,4) and (2,1)

$d = \sqrt{(2--4)^2 + (1-4)^2}$

$d = 6.71$ units

② State the gradient and y-intercept of:

a) $y = 2x - 5$

$m = 2$

y-int $= (0,-5)$

b) $y + 3x - 8 = 0$

$y = -3x + 8$

$m = -3$

y-int $= (0,8)$

c) $3y + 9 = x$

$y = -3 + \frac{x}{3}$

$m = \frac{1}{3}$

y-int $= (0,-3)$

d) $2x + 10 + 4y = 6$

$4y = -2x - 4$

$y = -\frac{x}{2} - 1$

$m = -\frac{1}{2}$

y-int $= (0,1)$

③ determine the equation between:

a) (2,-1) and (-3,5)

$m = \frac{5--1}{-3-2} = \frac{-6}{5}$

$y = \frac{-6}{5}x + c$

$-1 = \frac{-6}{5}(2) + c$

$c = -1 + 2.4$

$c = 1.4$

$\therefore y = \frac{-6}{5}x + 1.4$

b) (-1,6) and (0,2)

$m = \frac{2--6}{0--1} = \frac{-4}{1} = -4$

$y = -4x + c$

$y = -4x + 2$

④ determine the equation of the line perpendicular to $6y + 2x + 12 = 0$ that goes through $(2,-4)$

$6y = -2x - 12$

$y = \frac{-x}{3} - 2$

$m = \frac{1}{3}$

$\perp m = 3$

$y = 3x + c$

$-4 = 3(2) + c$

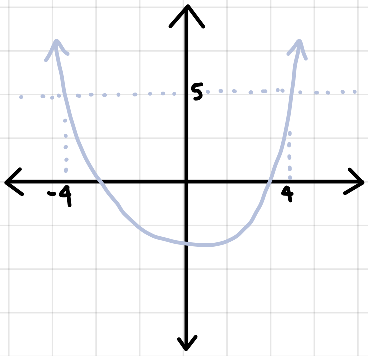
$c = -10$

$\therefore y = 3x - 10$

FUNCTIONS VS RELATIONS

a function (e.g quadratic, cubic, linear etc.) can have several x values that give the same y-value

FOR EXAMPLE

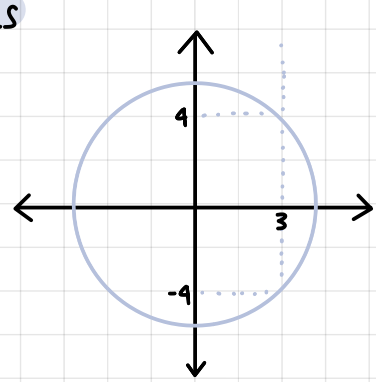


this is an example of a 'many-to-one' function

this shows that when $x = -4$ or when $x = 6$, y will equal 5

unlike a function, a relation (e.g circle, $y^2 = x$) can have one x-value that gives several different y-values

FOR EXAMPLE



this is an example of a 'one-to-many' relation

the method (shown in examples) used to determine relations from functions is called the 'vertical line test'

*note: if a function is 'one-to-one', it is also a function

FUNCTION NOTATION

- we say 'y is a function of x' because as x changes, y changes in response
  ↳ we can write this as $f(x)$ ('ef of ex') and simply put, it replaces 'y' in an equation
  - ☐ so $y = 3x - 2$ can be $f(x) = 3x - 2$
  - ☐ then we could show y equals when $x = 5$ as:
    $= f(5) = 3(5) - 2$
    $= 13$

to determine x when the function is equal to 19, we write:
$f(x) = 19$, $3x - 2 = 19$, $3x = 21$, $x = 7$

* determine a when $f(a) = 19$ bruh

SOLVING QUADRATICS

completing the square

Completing the Square

Solve Quadratics

1. If a ≠ 1, divide the quadratic by a.

2. Write the quadratic in the form

$$x^2 + bx = c$$

3. Add $(b/2)^2$ to both sides of the equation.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$$

4. Factor the left side of the equation into a perfect square.

$$\left(x + \frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$$

5. Square root both sides of the equation and solve for x.

$$x + \frac{b}{2} = \pm\sqrt{c + \left(\frac{b}{2}\right)^2}$$

TRANSFORMATIONS OF THE GENERAL FORM

CHANGES TO $f(x)$	NOTATION	TYPE OF TRANSFORMATION
Adding 'k' to the function	$f(x)+k$	- Means f(x) is translated k units vertically - If k>0 goes up - If k<0 goes down
Replacing x with x+/-k	$f(x-k)$ or $f(x+k)$	- Means f(x) is translated k units horizontally - If x+k -> moves k units left - If x-k -> moves k units right
If f(x) is multiplied by -1	$-f(x)$	- means f(x) is reflected in the x-axis
Replacing x with -x	$f(-x)$	- Means f(x) is reflected in the y-axis
Multiplying function by 'a' units	$af(x)$	- Means f(x) is dilated by a scale factor of a, parallel to the y axis - i.e. stretched vertically
Replacing x with ax	$f(ax)$	- means f(x) is dilated by a factor of $\frac{1}{a}$, parallel to the x-axis - i.e. stretch of compress horizontally

FACTORISING CUBICS

$\boxed{\text{if } y = ax^3 + bx^2 + cx}$ (or any combination of these 3 terms)

① Factor out 'x'
② Factorise quadratic using any method already learned

EXAMPLES

① Factorise:

a) $y = x^3 + 8x^2 + 12x$
$= x(x^2 + \overset{+}{8x} + \overset{\times}{12})$
$= (x)(x+2)(x+6)$

b) $y = -x^3 + 4x^2 + 5x$
$= -x(x^2 - \overset{+}{4x} - \overset{\times}{5})$

② Determine all axis intercepts of:

a) $y = 4x^3 - 6x$

y axis int, $x = 0$
$y = 4(0)^3 - 6(0)$
$= 0$ $(\therefore 0,0)$

x axis int, $y = 0$
$0 = 4x^3 - 6x$
$0 = (2x)(2x^2 - 3)$

$\therefore 2x = 0$, $x = 0$
and
$2x^2 - 3 = 0$, $2x^2 = 3$
$x^2 = \frac{3}{2}$, $x = \pm\sqrt{\frac{3}{2}}$

b) $y = 2x^3 - 12x^2 + 18x$

y-int: $(0,0)$
x axis int, $y=0$:
$0 = 2x^3 - 12x^2 + 18x$
$= 2x(x^2 - 6x + 9)$
$= 2x(x-3)^2$

$\therefore 2x = 0$, $x = 0$
$x - 3 = 0$, $x = 3$
$(x = 0)$ and $(x = 3)$
$(0,0)$ and $(3,0)$

$\boxed{\text{if } y = ax^3 + bx^2 + cx + d}$

① Determine one root of function first (note: sometimes the question is scaffolded to help you)
- try $x = 1$ ... if $y = 0$ when $x = 1$ then $(x-1)$ is a factor
$\therefore y = (x-1)(ax^2 + bx + c)$

② "by inspection" determine a and c of quadratic factor

③ Expand

④ Group coefficients of either 'x' (or 'x^2') and solve for c (or b). Use 'c' value from original expanded cubic

⑤ Now factorise quadratic factor of $(x-1)(ax^2+bx+c)$ + give final answer

EXAMPLES

① factorise $y=x^3-6x^2-x+6$

try $x=1$

$y=(1)^3-6(1)^2-(1)+6$

$=1-6-1+6$

$=0$ $\quad\therefore$ one root $=(1,0)$

*opposite sign

$\therefore y=(x-1)(ax^2+bx+c)$

$a=1$, $c=-6$

$y=(x-1)(x^2+bx-6)$

$=x^3+bx^2-6x-x^2-bx+6$

$=-6x-bx=-x$ $\quad$ or another way: $b+(-1)=-6$

$b=-6+1$ $\quad b=-5$

$-6-b=-1$

$-b=-1+6$

$\therefore b=-5$

what does b have to be?

$y=(x-1)(x^2-5x-6)$

$=(x-1)(x-6)(x+1)$

② $f(x)=x^3-10x^2+31x-30$

try $x=1$ a.k.a $f(1)$

$y=(1)^3-10(1)^2+31(1)-30$

$=1-10+31-30$

$=-8$ $\quad\therefore x=1$ is not on x-axis intercept

try $f(2)$

$y=(2)^3-10(2)^2+31(2)-30$

$=8-40+62-30$

$=0$ $\quad\therefore x=2$ is an x-axis intercept

opposite sign

$\therefore f(x)=(x-2)(ax^2+bx+c)$

a must $=1$ so it $=x^3$

what does c have to be to make -30 (from the top bit)

inspection:

$a=1$

$c=15$

now just b is left

$\therefore f(x)=(x-2)(x^2+bx+15)$

expand: $x^3+bx^3+15x-2x^2-2bx-30$

$b-2=-10$

use the coefficients + original coefficient

* check using coefficients: $15-2b=31$, $-2b=31-5$, $b=\frac{16}{-2}$, $b=-8$ $\quad\therefore$ correct

$\therefore f(x)=(x-2)(x^2-8x+15)$

$=(x-2)(x-3)(x-5)$

x-ints: $(2,0)(3,0)(5,0)$

REVIEW

① State the Domain + Range

a) $y = -x^2 + 4$

Domain: $\{x \in \mathbb{R}\}$ Range: $\{y \in \mathbb{R}, y \leq 4\}$ it is a max. +p, moved 4 units up

b) $y = \frac{1}{x} - 4$

Domain: $\{x \in \mathbb{R}, x \neq 0\}$ has an asymptote Range: $\{y \in \mathbb{R}, y \neq -4\}$ since x will never be 0 (the fraction will never $= 0$)

c) $f(x) = 3 - \sqrt{x-2}$

Domain: $\{x \in \mathbb{R}, x \geq 2\}$ Range: $\{y \in \mathbb{R}, y \leq 3\}$ is negative so its opposite sign

② consider the functions

$f(x) = 3x - 5$ $g(x) = 3x^2 - 2x + 1$ $h(x) = 2x + 4$

a) determine $f(6)$

$f(6) = 3(6) - 5$, $f(6) = 18 - 5$ $f(6) = 13$

b) determine k if $f(k) = -11$

$-11 = 3k - 5$, $-11 + 5 = 3k$, $-6 = 3k$, $\frac{-6}{3} = k$, $k = -2$

c) state the value of m if $h(m) = g(m)$

$2m + 4 = 3m^2 - 2m + 1$, $2m + 2m + 3 = 3m^2$ $0 = 3m^2 - 4m - 3$ need quadratic formula

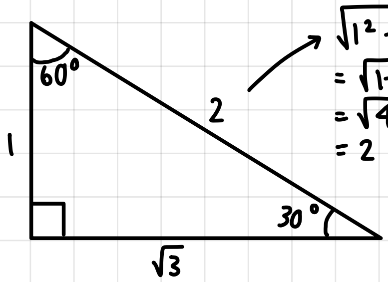
$m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-3)}}{2(3)} = \frac{4 \pm \sqrt{52}}{6}$

d) determine $h(2a+b)$ in terms of a and b

$= 2(2a+b) + 4$, $= 4a + 2b + 4$

EXACT VALUES – TRIG

classpad = on standard, not Decimal



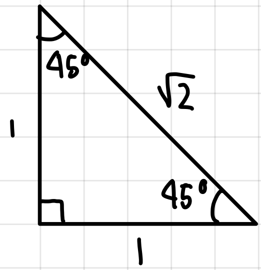
$\sqrt{1^2+\sqrt{3}^2}$
$= \sqrt{1+3}$
$= \sqrt{4}$
$= 2$

e.g - Decimal of sin (60)
= 0.8660254038...
Standard of sin (60)
$= \frac{\sqrt{3}}{2}$

find $\sin(30) = \frac{O}{H} = \frac{1}{2}$
find $\tan(30) = \frac{O}{A} = \frac{1}{\sqrt{3}}$

find $\sin(60) = \frac{\sqrt{3}}{2}$
find $\tan(60) = \frac{\sqrt{3}}{1}$

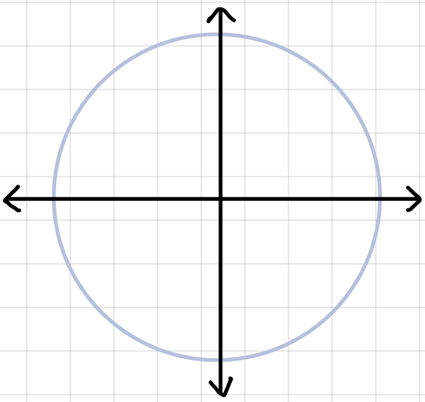
find $\cos(30) = \frac{A}{H} = \frac{\sqrt{3}}{2}$
find $\cos(60) = \frac{\sqrt{3}}{2}$



find $\sin(45) = \frac{O}{H} = \frac{1}{\sqrt{2}}$

find $\cos(45) = \frac{A}{H} = \frac{1}{\sqrt{2}}$

find $\tan(45) = \frac{O}{A} = \frac{1}{1} = 1$



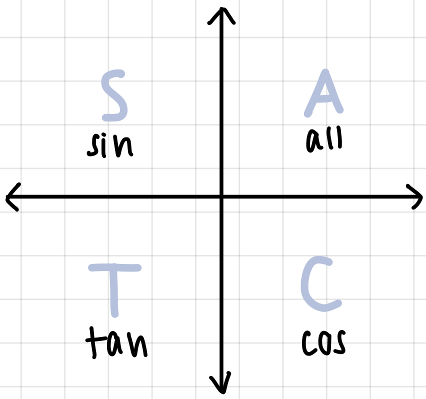
(x)
$\cos(0) = 1$
$\cos(90) = 0$
$\cos(180) = -1$

(y)
$\sin(0) = 0$
$\sin(90) = 1$
$\sin(180) = 0$

$\tan(0) = \frac{\sin(0)}{\cos(0)} = \frac{0}{1} = 0$
$\tan(90) = \frac{\sin(90)}{\cos(90)} = \frac{1}{0}$
= undefined

QUADRANTS

what values will be positive in the quadrants?



$\sin(150) = \sin(30)$

$\cos(150) = -\cos(30)$

$\tan(150) = -\tan(30) = \frac{-1}{\sqrt{3}}$

RADIANS

the number of radians in half a circle = π (i.e $180°$)
the number of radians in a full circle = 2π (i.e $360°$)

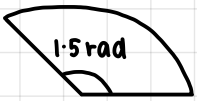
Converting degrees ⟶ radians

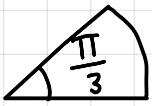
$$\boxed{\text{radians} = \text{degrees} \times \frac{\pi}{360}}$$

Converting radians ⟶ degrees

$$\boxed{\text{degrees} = \text{radians} \times \frac{180}{\pi}}$$

Radians can be given in decimal or exact form
e.g





some standards

$180° = \pi$
$90° = \frac{\pi}{2}$
$270° = \frac{3\pi}{2}$
$360° = 2\pi$
$30° = \frac{\pi}{6}$
$60° = \frac{\pi}{3}$
$45° = \frac{\pi}{4}$

CIRCULAR RELATIONS

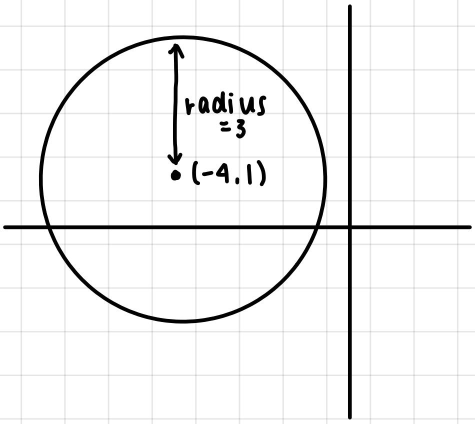
$\boxed{x^2+y^2=r^2}$

$\boxed{(x+a)^2+(x+b)^2=r^2}$ radius

flip the sign = midpoint co-ord

radius

$(x+4)^2+(y-1)^2=3^2$



EXPANDING + REARRANGING

$(x+4)^2+(y-1)^2=3^2$

$(x+4)(x+4)+(y-1)(y-1)=3^2$

$x^2+8x+16+y^2-2y+1=9$

$x^2+8x+y^2-2y=-8$

EXAMPLE 4a – exercise 7D

centre @ (3,5) and r=5

$(x-3)^2+(y-5)^2=5^2$

$x-6x+9+y^2-10y+25=25$

$x^2+y^2-6x-10y=25-25-9$

$x^2+y^2-6x-10y=-9$

$x^2+y^2+6y=10x$

$x^2-10x+25+y^2+6y+9=0+25+9$

completing the square

$(x-5)^2+(y+3)^2=34$

$=(\sqrt{34})^2$

∴ circle centre @ (5,-3)

radius = $\sqrt{34}$

$(x-2)^2+y^2=16$

$(x-2)^2+(y+0)^2=4^2$

BOOK QU

$x^2+y^2-6x+10y+25=0$

$x^2-6x+9+y^2+10y+25=9$

$(x-3)^2+(y+5)^2=3^2$

centre = (3,-5)

(-4,-3)

ANGLE SUM AND ANGLE DIFFERENCE